Experimental observation of characteristic relations of type-III intermittency in the presence of noise in a simple electronic circuit

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We investigate the characteristic relations of type-II and -III intermittencies in the presence of noise. The theoretically predicted characteristic relation is that $\langle \ell \rangle \sim \exp\{|\epsilon|^2\}$ for a negative regime of ϵ and $\langle \ell \rangle \sim \epsilon^{-\nu}$ for the positive regime of ϵ $(1/2 \leqslant \nu < 1)$, where $\langle \ell \rangle$ is the average laminar length and $(1+\epsilon)$ is the slope of the local Poincaré map around the tangent point. We experimentally confirm these relations in a simple electronic circuit.

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I. INTRODUCTION

Intermittency is one of the main routes of the transition to chaos in nonlinear dynamical systems along with period doubling, quasiperiodicity, and crises. The phenomenon is characterized by the irregular changes between long quasiregular signals, so-called laminar phase, and relatively short period of chaotic bursts.

According to Pomeau and Manneville, intermittency can be classified into three types [1], depending on their local geometry of the manifolds (local Poincaré map): type-I intermittency for quadratic structures and type-II and -III intermittencies for cubic ones. They showed that the length of the laminar ℓ, which is defined as the time interval between the bursts, depends on the initial value, and the average laminar length $\langle \ell \rangle$ is a statistical quantity that depends on the local Poincaré map as well as reinjection probability distribution (RPD) [2]. Since noise is unavoidable in nature, the intermittency with noise is of fundamental importance in the practical application of many topics in science. Eckmann, Thomas, and Wittwer [3] studied intermittency analytically with added noise in the neighborhood of the intermittency threshold. Even though they treated the problem rigorously, they only considered quadratic local structures of manifolds, i.e., type-I intermittency, with a positive channel width δ (which is defined as the distance between the quadratic manifold and the diagonal line). The recent study has explicitly shown that the characteristic relation of type-I intermittency with added noise changes nontrivially as the parameter δ changes from the positive to negative [4]; namely, it is deformed from $\langle \ell \rangle \propto \delta^{-1/2}$ to $\langle \ell \rangle \propto \exp\{|\delta|^{3/2}\}$. Very recently, this relation has been experimentally confirmed in electronic circuits [5].

In contrast that many researches on the study of type-I intermittency have been done so far, those of type-II and -III intermittencies are rarely studied in spite of the importance of their local geometry of the manifolds, especially in the presence of noise [6,7]. Pikovsky obtained the characteristic relation of type-II intermittency with added noise in the form of power series by the use of Fokker-Planck equation (FPE) under the assumption of a symmetrical mapping and uniform RPD [8]. The relations are $\langle \ell \rangle \propto \frac{1}{2}\epsilon^{-1/2}$ for $\epsilon \gg 0$ and $\langle \ell \rangle \propto \pi |2\epsilon|^{-1/2} \exp\{\frac{1}{2}|\epsilon|^2\}$ for $\epsilon \ll 0$, where $(1+\epsilon)$ is the slope of the local Poincaré map around the tangent point. Up to now, however, there is no experimental investigation of the characteristic relations of type-II and -III intermittencies in the presence of noise.

In this paper, we experimentally investigate the characteristic relations of the average laminar length near the threshold of type-II and -III intermittencies in the presence of noise. The paper is organized as follows. In Sec. II, the characteristic relations for positive and negative ϵ are reviewed and derived analytically. Two different approaches are applied for the cases of positive and negative ϵ. While the consideration of positive ϵ is straightforward, negative ϵ is considered by using the FPE. In addition, we discuss RPD which is another important factor to determine the characteristic relations of type-II and -III intermittencies [2,9]. In Sec. III the description of the experimental setup, which is a simple inductor-resistor-diode (LRD) circuit, is presented. In Sec. IV, the experimental results and discussions are presented. Finally, the main results of the paper are summarized in Sec. V.

II. THEORETICAL ANALYSIS

The local Poincaré maps of type-II and -III intermittencies in the presence of noise are described by the following well-known difference equation [1,8,10]:

$$x_{n+1} = \pm(1+\epsilon)x_n \pm a x_n^3 + \sqrt{2D}\xi_n, \tag{1}$$

where a is the positive arbitrary constant, ϵ is the parameter, and D is the dispersion of Gaussian noise ξ_n. The signs of linear and cubic terms in x correspond to type-II intermittency for the plus and type-III intermittency for the minus, respectively. Since the local Poincaré map of type-III intermittency can be described the same as that of type-II intermittency, it is quite enough to discuss the characteristic relation of type II without loss of generality.

The characteristic relations can be obtained for two cases depending on the sign of ϵ, where $(1+\epsilon)$ is the slope of the local Poincaré map. They correspond to $\epsilon > 0$ and $\epsilon < 0$ as shown in Fig. 1. For $0 < D \ll \epsilon$, noise effect can be neglected

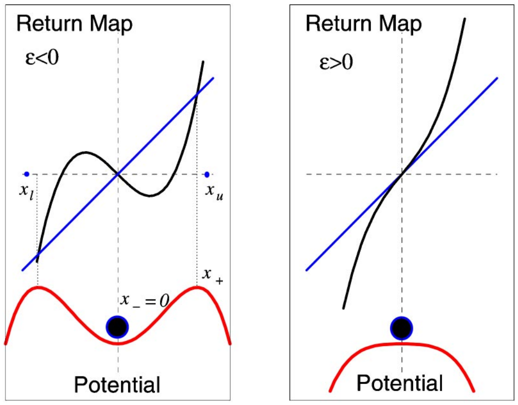

FIG. 1. Schematic relation between the return map and the potential from a mechanical analogy where (a) $\epsilon<0$ and (b) $\epsilon>0$. Fixed points of the return map correspond to extremal positions of the potential.

since the slope mainly contributes to the characteristic relation. So the characteristic relation can be obtained by solving the equation of a general form of type-II intermittency $x_{n+1}=(1+\epsilon)x_n+ax_n^3$; whereas for $\epsilon<0$, Eq. (1) can be transformed into a differential form to solve FPE.

A. Characteristic relation for $\epsilon>0$

When we neglect the effect of noise for $0<D\ll\epsilon$, the characteristic relation of type-II intermittency can be obtained by integrating $dx/dt=ax^3+\epsilon x$, where $dx/dt\approx x_{n+1}-x_n$ under the long laminar length approximation. If we give a gate which sets an acceptance $|y_{in}|\leqslant c$ on deviations in the laminar region, the laminar length $\ell(y_{in},c)$ for the reinjection at y_{in} becomes

$$\ell(y_{in},c)=\int_{y_{in}}^{c}\frac{dx}{ax^3+\epsilon x}=\frac{1}{2\epsilon}\left[2\ln\left(\frac{c}{y_{in}}\right)-\ln\left(\frac{ac^2+\epsilon}{ay_{in}^2+\epsilon}\right)\right].$$

Then, the average laminar length is given as follows:

$$\langle\ell\rangle=\int_{\Delta}^{c}\ell(y_{in},c)\,P(y_{in})dy_{in},$$

where Δ is the value of y_{in} representing the lower bound of the reinjection.

From the above equation, we can obtain various characteristic relations according to the RPD that is given by $P(y_{in})$. When the lower bound of reinjection is fixed near the tangent point, we can obtain the characteristic relation of the form $\langle\ell\rangle\propto\epsilon^{-1}$ as $\epsilon\to0$. For the uniform reinjection, we can obtain the characteristic relation of the form $\langle\ell\rangle\propto\epsilon^{-1/2}$ as $\epsilon\to0$; and for the nonuniform reinjection of the form $1/\sqrt{y_{in}-\Delta}$, the characteristic relation is $\langle\ell\rangle\propto\epsilon^{-3/4}$ as $\epsilon\to0$. It is evident from these results that the characteristic relation strongly depends on the RPD for $0<D\ll\epsilon$, even if noise is applied.

B. Characteristic relation for $\epsilon<0$

In the absence of noise and for $\epsilon<0$, the return map of Eq. (1) has one stable fixed point at the origin and two unstable fixed points. A trajectory is attracted to the stable fixed point, and the system never shows intermittent behaviors. However, added noise changes the situation drastically since the particle can escape from the well. In this case, we can calculate the average laminar length for a given noise amplitude. So, the problem becomes nontrivial. We are interested in the characteristic relations depending on the RPD.

In the long laminar region, Eq. (1) can be approximated to a stochastic differential equation as follows [11]:

$$\dot{x}=-V'(x)+\sqrt{2D}\xi(t), \tag{2}$$

where the dot and the prime denote the differentiation with respect to t and x, respectively. Here $\xi(t)$ is the *Gaussian white noise* such that $\langle\xi(t')\xi(t)\rangle=\delta(t'-t)$ and $\langle\xi(t)\rangle=0$ [12], and $V(x)$ is the potential given by $V(x)=-\frac{1}{2}\epsilon x^2-\frac{1}{4}ax^4+C$, where C is the integration constant. We can regard the above equation as the equation of motion of a massless point particle under the potential $V(x)$ with Gaussian random perturbation $\xi(t)$.

We can convert the stochastic differential equation into the FPE type. Then, the scaling of the average laminar length can be estimated by the solution of the FPE. It is often required to know how long a particle whose position is described by a FPE remains in a certain region of x. The solution of this problem can be obtained by the use of *backward Fokker-Planck equations* [11,12].

$$\frac{\partial G(x,t)}{\partial t}=-V'(x)\frac{\partial G(x,t)}{\partial x}+D\frac{\partial^2 G(x,t)}{\partial x^2}, \tag{3}$$

where $G(x,t)$ is the probability density of a particle at $\{x,t\}$. The *mean first passage time* (MFPT) function $T(x)=\langle t\rangle=-\int_0^\infty t[\partial G(x,t)/\partial t]dt$ is defined as the average transition time from the reinjection to the escaping point x due to the backward property under the potential $V(x)$ and random perturbation. In this case, the MFPT function $T(x)$ can be approximated as follows [11,12]:

$$-1=-V'(x)\frac{dT}{dx}+D\frac{d^2T}{dx^2}, \tag{4}$$

where we have used boundary conditions $G(x_0,0)=1$ and $\lim_{t\to\infty}G(x,t)=0$; here $G(x_0,0)=1$ implies that the initial particle position is x_0. The MFPT function $T(x)$ is the average transition time from x_0 to the escaping point x under the potential $V(x)$ and random perturbation.

One can verify easily that the solution of Eq. (4) is given by the expression

$$T(x)=c_1\int_{x_l}^{x}dx'\exp\left\{\frac{1}{D}V(x')\right\}-\frac{1}{D}\int_{x_l}^{x}dx'\int_{x_l}^{x'}dx''\exp\left\{\frac{1}{D}[V(x')-V(x'')]\right\}, \tag{5}$$

where c_1 is the integration constant, x_l is the lower bound of the laminar phase, and x is the destination point of the transition.

If noise is small enough, that is, $D \ll 1$, the first term in the above equation is suppressed by the factor of $1/D$ and the second term becomes dominant. Since the second term is not integrable analytically, we expand the potential near the extremal point $x_\pm$ (see Fig.1) approximately such that $V(x) \approx V(x_\pm) + [V''(x_\pm)/2](x-x_\pm)^2 + O((x-x_\pm)^3)$.

In that case, the MFPT function $T(x)$ can be approximated for $\epsilon \ll 0$ as follows:

$$T(x) \approx -\frac{1}{D}\exp\left\{\frac{1}{D}[V(x_+)-V(x_-)]\right\} \times \int_{x_l}^{x} dx' \int_{x_l}^{x'} dx'' \exp\left\{\frac{1}{2D}[V''(x_+)(x'-x_+)^2 - V''(x_-)(x''-x_-)^2]\right\}. \quad (6)$$

The extremal points are given by $x_+ = \sqrt{-\epsilon/a}$ and $x_- = 0$ in Eq. (2). We can perform the integration of the quadratic exponent [12] and then obtain the following approximated solution of the MFPT equation, as we take the limit $x \to \infty$ and $x_l \ll x_-$:

$$|T(x)| \sim \frac{\sqrt{2\pi}}{|\epsilon|}\exp\left\{\frac{|\epsilon|^2}{4aD}\right\}, \quad \epsilon < 0. \quad (7)$$

Therefore, $|T(x)| \sim \exp(\epsilon^2/4aD)$ for $\epsilon \ll 0$.

C. Characteristic relations for uniform reinjection

So far, our calculation is done with *fixed* RPD, i.e., $P(x_{in}) = \delta(x_{in}-\Delta)$. (In all of our simulations we set the reinjection point at $\Delta = 0$.) The RPD $P(x_{in})$ is also an important factor which affects the scaling relation of the average laminar length. Now, we consider the *uniform* RPD case. The equation of average laminar length $\langle \ell \rangle$, which is the average time interval between the bursts [13,14], can be given by the equation below, generically:

$$\langle \ell \rangle = \int_{x_l}^{x_u} dx' P(x')\, \ell(x'), \quad (8)$$

where the lower bound reinjection point is x_l, the upper bound is x_u, and $\ell(x')$ is the laminar length reinjected at x'. RPD, $P(x')$, is normalized as $\int_{x_l}^{x_u} dx' P(x') = 1$. Using $T(x)$, average laminar length under consideration of RPD is given approximately by the following equation:

$$\langle \ell \rangle = |T(0)| - \langle \ell \rangle_A,$$

$$\langle \ell \rangle_A = D^{-1}\int_0^A dx' P(x') \int_0^{x'} dx e^{f(x)/D} \int_0^x dy\, e^{-f(y)/D}, \quad (9)$$

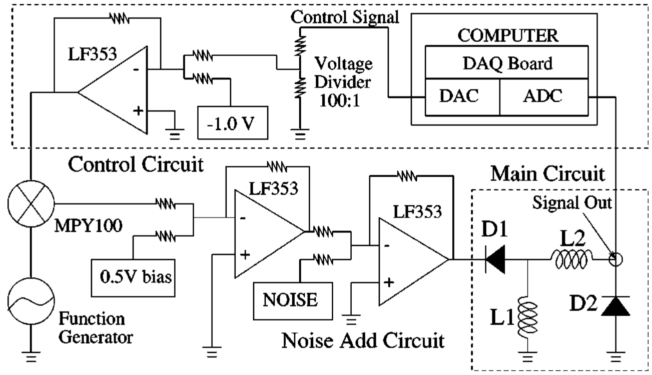

FIG. 2. Schematic diagram of experimental setup where DAC and ADC are digital-analog converter and analog-digital converter, respectively. The control signal comes from the DAC.

where $|T(0)|$ is the average transition time from the injection point $x=0$, $f(x)$ a potential near $V(x_+)$, and $f(y)$ a potential near $V(x_- = 0)$ in Fig. 1. We can therefore expand $f(y)$ as $V(x)$ in the vicinity of $x=0$. If the reinjection is uniform, we can give $P(x') = 1/A$, where A is the available region of reinjection. If we take the limit $A \to \infty$, the exponential terms will go to zero faster than any other growing terms. Therefore average laminar length in uniform RPD reads

$$\langle \ell \rangle \sim |T(0)| = \langle \ell \rangle_F, \quad \epsilon < 0, \quad (10)$$

where the subscript F means reinjection of fixed point at $x = 0$. Equation (10) shows that in the negative region of ϵ, the average laminar lengths obtained by uniform RPD and that of fixed RPD are the same.

III. EXPERIMENTAL SETUP

The schematic diagram of the experimental setup is shown in Fig. 2. The circuit consists of two inductors, each of which has inductance 100 mH and dc resistance 130 Ω, and two silicon junction diodes (1N4007). A dc voltage in the range between ±10 V from a digital-analog (DA) converter (National Instruments PCI-MIO-16E-1) is reduced to 1/100 with a voltage divider, and the reduced voltage is added to another −1.0 V dc voltage. Sinusoidal signals from a function generator (Tektronix, FG 501A) is multiplied by the total dc voltage with a multiplier (MPY100). The frequency and the amplitude of the sinusoidal signals are fixed at 18.0 kHz and 3.0 V, respectively. We add 0.5 V bias voltage and random noise signals from a random signal generator (HP 33120A) to the total sinusoidal signals. The amplitude of noise is fixed at 0.1 V. The total signals are applied to LRD circuit. All the external signals are added by using operational amplifiers (LF353). Through this configuration, we can vary the amplitude of the driving signal precisely by 0.03 mV at each step by the help of a 12-bit resolution DA converter.

The voltage across the second diode, D2, is stored in a personal computer through an analog-digital (AD) converter that is installed in the same board as the DA converter. The AD converter, of which digitizing time is 8 μsec, can measure the experimental data without distortion. The AD and

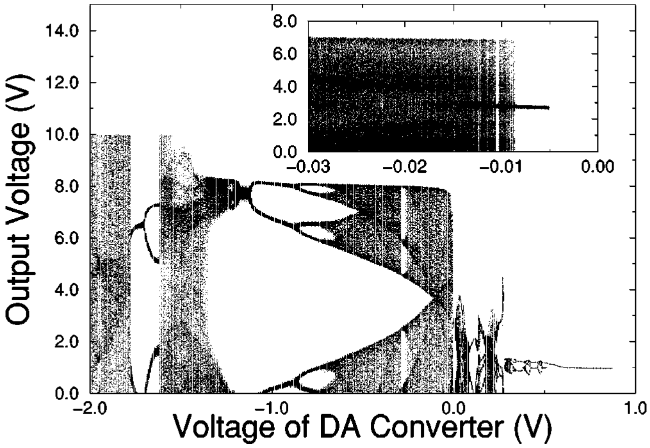

FIG. 3. Bifurcation diagram in the absence of noise according to the voltage from the DA converter in the region from 1.0 V to −2.0 V. The region corresponds to the range of the amplitude of the driving signal from 0.0 V to 9.0 V. The inset is the enlarged region where type-III intermittency appears.

DA converters are controlled with the LABVIEW program. The obtained data are also analyzed with the LABVIEW program. The chaotic outputs and intermittent behaviors are monitored with a digital storage oscilloscope (LeCroy 9310) simultaneously, of which sampling time is 0.5 nsec and the memory size is 2 Mbyte.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

The circuit exhibits various nonlinear dynamical behaviors due to the nonlinear capacitance of the junction in the diodes [15]. First, we have obtained a bifurcation diagram depending on the amplitude of the driving signal after the noise signal is turned off. The amplitude of the driving signal is controlled with a computer through the DA converter. To obtain the data, we measure the peak voltages of 500 cycles at each input voltage of the DA converter with the LABVIEW program. In the bifurcation diagram, since the detecting voltage of the AD converter is less than 10.0 V, the voltage higher than 10.0 V is omitted. The bifurcation diagram is shown in Fig. 3. Here we can observe period-doubling bifurcation, chaos, and periodic windows as we increase the voltage from the DA converter. We can also observe typical temporal behaviors of type-III intermittency when the voltage from the DA converter is around zero (the amplitude of the driving signal is around 3.0 V). The inset is the enlarged bifurcation diagram near the region of type-III intermittency. This figure shows that the chaotic band transits to a period-2 window, and the maximum amplitude of the signal drops from 7.2 V to 2.8 V.

The temporal behaviors of the circuit are shown in Figs. 4(a) and 4(b) which correspond to the voltage of the DA converter of -0.012 V and -0.010 V, respectively. In the figures, almost regular rectified signals across the diode are interrupted irregularly by chaotic bursts. When the voltage of the DA converter is -0.012 V the circuit exhibits short periods of regular rectified signals whose amplitude is about 2.8 V. On the other hand, it exhibits long period of regular

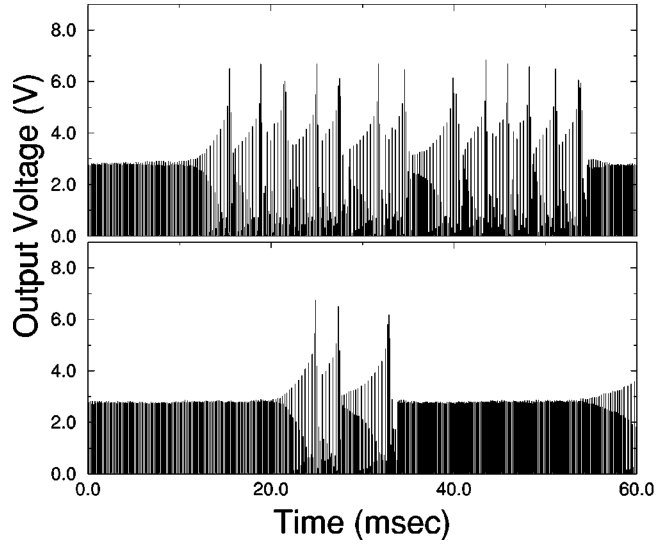

FIG. 4. Temporal behaviors of type-III intermittency in the absence of noise for (a) -0.012 V and (b) -0.010 V of the DA converter.

signals for -0.010 V. Notice that longer laminar phases appear as the amplitude of the driving signal reduces.

From the time series of the data, we obtain the x_n versus x_{n+2} return map as shown in Fig. 5(a) when the voltage of the DA converter is -0.012 V. Although the return map crosses the diagonal line, it is hard to find the local Poincaré map of type-III intermittency, $x_{n+1}=-(1+\epsilon)x_n-x_n^3$. So, we obtain the return map of x_n versus x_{n+4} as shown in Fig. 5(b). The figure shows the feature of type-II intermittency. The return map is well fitted by the cubic curve of form $(1+0.1)x+x^3$ when we translate the map to the tangent point, i.e., $x-2.8$ where the tangent point is about 2.8 V. Based on this observation, we can conclude that the return map of Fig. 5(a) implies the local Poincaré map of type-III intermittency.

In order to obtain the characteristic relation of type-III intermittency with added noise, we first determine the bifurcation point V_t from the bifurcation diagram of the inset in Fig. 3 by searching the last point where chaotic burst appears. The determined bifurcation point from the DA converter is $V_t=-8.5$ mV. Noise of amplitude 0.1 V is added to the circuit (see Fig. 2). This noise is δ correlated and has a Gaussian profile [5]. Furthermore, we experimentally

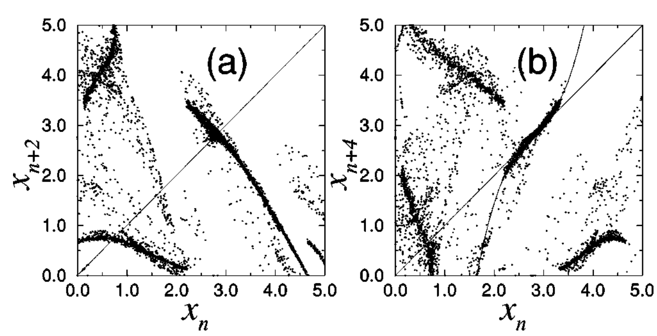

FIG. 5. Return maps of (a) x_n vs x_{n+2} and (b) x_n vs x_{n+4} when the voltage of the DA converter is -0.012 V. (b) Typical local Poincaré map of type-II intermittency.

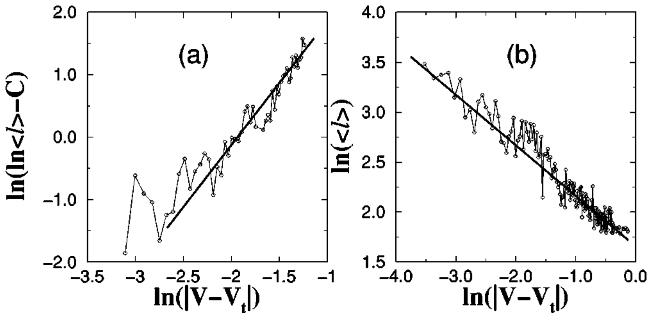

FIG. 6. The characteristic relations of the average laminar length vs $|V-V_t|$ in the presence of 0.1 V noise (a) for $\epsilon<0$ and (b) for $\epsilon>0$, where C is the y interception of $\ln\langle l_0\rangle$ that is 3.7.

found that RPD is almost uniform around the tangent point [5]. The uniform tendency of RPD could depend on the profiles of external noise and the overlapping of reinjections from left and right sides due to the global structure of the map. We measure the average laminar length depending on the voltage from the DA converter. In this experiment, we increase the dc voltage from the DA converter by 0.01 mV at each step for a fine tuning. As the voltage from the DA converter increases from negative voltage, the length of the laminar is measured by counting the peaks of regular signals. In this way, we are able to measure the laminar length up to 1.2×10^5, which is the limit of computer memory, by using the LABVIEW program.

Figure 6 shows the characteristic relations for two regions $V_t-V<0$ ($\epsilon<0$) and $V_t-V>0$ ($\epsilon>0$). For $\epsilon<0$, the plot of $\ln(\ln\langle\ell\rangle-\ln\langle\ell_0\rangle)$ versus $\ln|V_t-V|$ is well fitted to the slope of 2 as shown in Fig. 6(a). Here, we obtain the y interception of $\ln\langle\ell_0\rangle$ that is 3.7 from linear regression of the curve appearing on the plot of $\ln\langle\ell\rangle$ versus $|V_t-V|^2$. It means that the characteristic relation obtained in the experiment agrees well with the theoretical one, i.e., $\langle\ell\rangle\propto\langle\ell_0\rangle\exp(\alpha|V-V_t|^2)$. We also obtain the slope of the average laminar length for $\epsilon>0$ on the plot of $\ln\langle\ell\rangle$ versus $\ln(V_t-V)$ as shown in Fig. 6(b). The slope of the experimental data is well fitted to the $-1/2$ slope of the solid line. This means that the characteristic relation is $\langle\ell\rangle\propto\epsilon^{-1/2}$ for $V\ll V_t$. This is the very characteristic relation of type-III intermittency for $0<D\ll\epsilon$ when RPD is uniform. This is well matched with the theoretical result of Sec. II. These results indicate that the characteristic relation deforms from $\langle\ell\rangle\propto\epsilon^{-1/2}$ for $\epsilon\gg0$ to $\langle\ell\rangle\propto\langle\ell_0\rangle\exp(\alpha|\epsilon|^2)$ for $\epsilon<0$.

V. CONCLUSION

For a positive ϵ and the uniform RPD, the theoretically predicted characteristic relations of type-II and -III intermittencies with added noise have the form $\langle\ell\rangle\sim\epsilon^{-1/2}$. However, the relation deforms as parameter ϵ moves to the negative regime. Then, the characteristic relation becomes $\langle\ell\rangle\sim\exp(|\epsilon|^2/4aD)$. From the electronic circiut experiment, we have obtained that the characteristic relation is $\langle\ell\rangle\propto\epsilon^{-1/2}$ for $\epsilon>0$ and $\langle\ell\rangle\propto\exp|\epsilon|^2$ for $\epsilon<0$. These results show that our experimental observations are in good agreement with the theoretical predictions for type-II and -III intermittencies in the presence of noise.

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[6] The importance of the basic structure of a local manifold can be easily illustrated by the well known catastrophe theory. According to Whitney's singularity theory [7], which constitutes the basis of the catastrophe theory, on a mapping of a surface onto a plane, one can only encounter two singularities, each of which is a *fold* and a *cusp*. These correspond to two fundamental manifolds of a local Poincaré map at the bifurcation point. It was proved that every singularity of a smooth mapping of a surface onto a plane, after appropriate small perturbation, splits into folds and cusps. This tells us that type-I, -II, and -III intermittencies are the only possible types when noise is involved, i.e., the structually stable ones.
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